

ENGINEERING
ADMISSIONS ASSESSMENT

D564/31

November 2020

60 minutes

SECTION 1

INSTRUCTIONS TO CANDIDATES

Please read these instructions carefully, but do not open this question paper until you are told that you may do so. This paper is Section 1 of 2.

A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and name.

At the end of 60 minutes, your supervisor will collect this question paper and answer sheet before giving out Section 2.

This paper contains **two** parts, **A** and **B**, and you should attempt **both** parts.

Part A Mathematics and Physics (20 questions)

Part B Advanced Mathematics and Advanced Physics (20 questions)

You are **strongly** advised to divide your time equally between the two parts: 30 minutes on **Part A** and 30 minutes on **Part B**. The scores for Part A and Part B are reported separately.

This paper contains 40 multiple-choice questions. There are no penalties for incorrect responses, only marks for correct answers, so you should attempt **all** 40 questions. Each question is worth one mark.

For each question, choose the **one** option you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

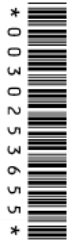
You **must** complete the answer sheet within the time limit.

You can use the question paper for rough working, but **no extra paper** is allowed. Only your responses on the answer sheet will be marked.

Dictionaries and calculators are NOT permitted.

Please wait to be told you may begin before turning this page.

This question paper consists of 34 printed pages and 2 blank pages.



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PART A Mathematics and Physics

- 1 A soldering iron has a copper tip of mass 2.0 g.

The tip is heated with 30 W of thermal power. In 50 s, the temperature of the tip increases by 200 °C.

How much energy is transferred from the tip to the surroundings in this time?

(specific heat capacity of copper = 400 J kg⁻¹ °C⁻¹)

A 160 J

B 500 J

☒ C 1340 J

D 1500 J

E 1660 J

F 1840 J

G 2500 J

$$\text{Thermal Power in} = 30 \text{ W}$$

$$\text{Energy} = 30 \text{ W} \times 50 \text{ s} = 1500 \text{ J}$$

$$\begin{aligned} E_{\text{cu}} &= mc\Delta T = 0.002 \times 400 \times 200 \\ &= 80000 \times 2 \times 10^{-3} \\ &= 160 \text{ J} \end{aligned}$$

$$1500 - 160 = 1340$$

- 2 The admission charge to a cinema is different for adults and children.

Admission for 2 adults and 3 children costs £20.

Admission for 4 adults and 4 children costs £34.

What does admission cost for 6 adults and 2 children?

A £27

B £29

C £33

☒ D £39

E £44

F £48

G £72

$$\begin{aligned} 2x + 3y &= 20 \\ \times 2 \quad \left(\begin{aligned} 4x + 6y &= 34 \\ 4x + 6y &= 40 \end{aligned} \right. \end{aligned}$$

$$2y = 6 \therefore y = 3$$

$$2x + 9 = 20 \therefore x = 5.5$$

$$6(5.5) + 2(3) = 33 + 6 = 39$$

- 3 Uranium-238 ($^{238}_{92}\text{U}$) decays by a series of alpha and beta (β^-) emissions to become the stable isotope lead-206 ($^{206}_{82}\text{Pb}$).

How many beta (β^-) particles are emitted in the decay of one uranium-238 nucleus to lead-206?

(A) 6

B 8

C 10

D 12

E 14

F 16

$$238 - 206 = 32$$

$$\frac{32}{4} = 8 \alpha \text{ decays}$$

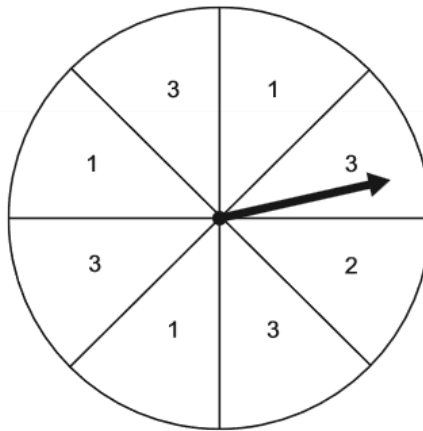
$$\text{After } 8 \times \alpha, Z_{\text{new}} = 92 - 8(2)$$

$$= 92 - 16 = 76$$

$$82 - 76 = 6 \therefore \text{There must be } 6 \beta^- \text{ decays}$$

- 4 A fair spinner has eight equal sections.

Each section has one number written on it, as shown.



The spinner is spun twice, and the two numbers scored are added.

What is the probability that the sum of the two numbers is 5? 3, 2 or 2, 3

- ☒ A $\frac{1}{8}$
B $\frac{5}{8}$
C $\frac{1}{16}$
D $\frac{3}{16}$
E $\frac{25}{64}$
F $\frac{55}{64}$

$$\begin{aligned} &= \frac{1}{2} \times \frac{1}{8} + \frac{1}{8} \times \frac{1}{2} \\ &= \frac{2}{8} = \frac{1}{4} \end{aligned}$$

- 5 A dc electricity transmission system uses an undersea cable to send electricity from one country to another. On a particular day, the first country supplies electricity at a voltage of 400 kV and 2000 A to the transmission system. The second country receives electricity from the transmission system at 160 kV and 4000 A.

What is the percentage efficiency of the system and how much energy is wasted every minute?

	efficiency %	energy wasted every minute / J
A	20	9.6×10^9
B	20	3.84×10^{10}
C	20	4.8×10^{10}
D	80	9.6×10^9
E	80	3.84×10^{10}
F	80	4.8×10^{10}

$$\text{Efficiency} = \frac{160 \times 4000^2}{400 \times 2000^2} = 80\%$$

$$\text{Power wasted} = 800000 \text{ kVA} - 640000 \text{ kVA}$$

- 6 Consider the four lines with the following equations.

$$\begin{aligned} \textcircled{1} \quad 2x + 6y &= 3 \rightarrow y = -\frac{1}{3}x + \frac{1}{2} \\ 2 \quad 9y &= 3x - 4 \rightarrow y = \frac{1}{3}x - \frac{4}{9} \\ \textcircled{3} \quad 2y &= 6x + 3 \rightarrow y = 3x + \frac{3}{2} \\ 4 \quad 4x + 6y - 9 &= 0 \rightarrow y = -\frac{2}{3}x + \frac{3}{2} \end{aligned}$$

$$\begin{aligned} &= 160000 \text{ kW} \\ 160000 \times 10^3 \times 60 \\ &= 9.6 \times 10^9 \text{ J} // \end{aligned}$$

Which two lines are perpendicular?

A 1 and 2

B 1 and 3

C 1 and 4

D 2 and 3

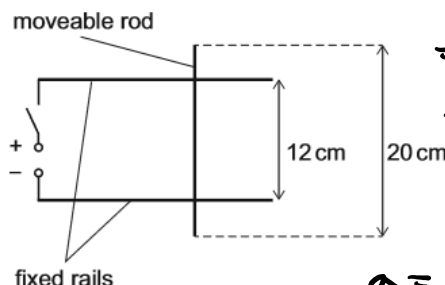
E 2 and 4

F 3 and 4

- 7 Two fixed horizontal metal rails are side by side and 12 cm apart. The rails are connected to a dc power supply by a switch that is initially open.

A freely moveable metal rod of length 20 cm is placed on the rails as shown in the diagram. The diagram shows the arrangement seen from above.

The angle between the rod and the rails is 90° .



$$F = BIL \sin \theta \quad \sin 90^\circ = 1$$

$$= 0.5 \times 2.4 \times 0.12 \times 1$$

$$= 1.2 \times 0.12$$

$$= 0.144 \text{ N}$$

$$a = \frac{F}{m} = \frac{F_g}{W} = \frac{0.144 \times 10}{0.4}$$

The whole arrangement is placed in a uniform magnetic field of magnitude 0.50 T that is directed perpendicularly into the page.

$$= 3.6 \text{ ms}^{-2}$$

The moveable rod has a weight of 0.40 N.

The switch is now closed. As a result, there is a current of 2.4 A in the circuit and the rod moves.

What is the initial magnitude of the acceleration of the rod and what is its direction?

(gravitational field strength = 10 N kg^{-1})

Fleming's
LH Rule

	acceleration / m s^{-2}	direction
A	0.36	to the left
B	0.36	to the right
C	0.60	to the left
D	0.60	to the right
E	3.6	to the left
F	3.6	to the right
G	6.0	to the left
H	6.0	to the right

- 8 Find the sum of the solutions of

$$2\left(\frac{x}{4}+3\right)^2 - \left(\frac{x}{4}+3\right) - 36 = 0$$

- A 2
B $\frac{3}{2}$
C $\frac{1}{2}$
D -4
E -13
F -22
G -26
H -34

$$\text{Let } v = \frac{x}{4} + 3$$

$$2v^2 - v - 36 = 0$$

$$2v^2 + 8v - 9v - 36 = 0$$

$$2v(v+4) - 9(v+4) = 0$$

$$v = -4 \quad \text{OR} \quad v = \frac{9}{2}$$

$$\frac{x}{4} = -7 \quad \text{OR} \quad \frac{x}{4} = \frac{9}{2}$$

$$x = -28 \quad \text{OR} \quad x = 6$$

$$-28 + 6 = -22$$

- 9 Two trolleys are moving towards each other along a straight horizontal track.

One trolley has mass 8.0 kg and is travelling to the right at 4.0 m s^{-1} .

The other trolley has mass 2.0 kg and is travelling to the left at 1.0 m s^{-1} .

When the trolleys collide they stick together.

How much kinetic energy is transferred to other forms of energy in the collision?

- A 2.0 J
B 18 J
C 20 J
D 28 J
E 35 J
F 40 J
G 45 J
H 65 J

COM:

$$8(4) + 2(-1) = 10(v)$$

$$v = 3 \text{ m s}^{-1} \rightarrow$$

$$\Delta KE = \frac{1}{2}(8)(4^2) + \frac{1}{2}(2)(1^2) - \frac{1}{2}(10)(3^2)$$

$$= 64 + 1 - 45 = \underline{\underline{20 \text{ J}}}$$

- 10 When the expression

$$(2x + 3)^2 - (x - 3)^2$$

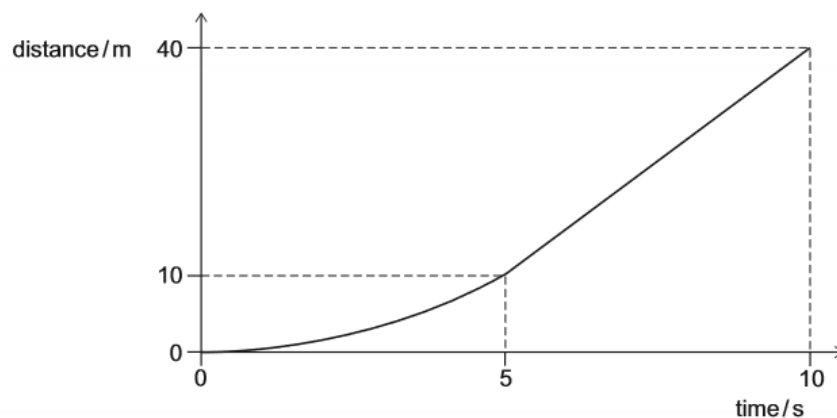
is written in the form $p(x + q)^2 + r$, where p , q and r are constants, what is the value of r ?

- ☒ A -27
B -9
C 0
D 3
E 15

$$\begin{aligned} &= 4x^2 + 12x + 9 - x^2 + 6x - 9 \\ &= 3x^2 + 18x \\ &= 3(x^2 + 6x) \\ &= 3[(x+3)^2 - 9] \\ &= 3(x+3)^2 - 27 \end{aligned}$$

- 11 A car of mass 800 kg travels in a straight line along a horizontal road.

The car accelerates **non-uniformly** from rest for 5.0 seconds and then moves at constant speed, as shown in the distance–time graph:



What is the average resultant force acting on the car over the time for which it is accelerating?

- A 320 N
B 480 N
C 640 N
D 960 N
E 1600 N
F 3200 N
G 4800 N

$$\text{Constant Speed} = \frac{40-10}{10-5} = 6 \text{ ms}^{-1}$$

$$\text{avg acceleration} = \frac{6-0}{5} = 1.2 \text{ ms}^{-2}$$

$$F_{\text{avg}} = ma_{\text{avg}} = 800 \times 1.2 = \underline{\underline{960 \text{ N}}}$$

- 12 The number of pairs of winter boots sold on a day is inversely proportional to the cube of the outside temperature on that day, measured in °C.

On a day when the outside temperature is 8 °C, 250 pairs of boots are sold.

The next day, when the outside temperature is x °C, the number of pairs of boots sold is 700% more than on the previous day.

What is the value of x ?

A 2

☒ B 4

C $\frac{8}{\sqrt[3]{7}}$

D $8\sqrt[3]{7}$

E 16

$$n \propto \frac{1}{T^3} \therefore n = \frac{k}{T^3}$$

$$250 = \frac{k}{8^3} \therefore k = 8^3 \times 250$$

$$n_{\text{next day}} = 8 \times 250$$

$$8 \times 250 = \frac{8^3 \times 250}{x^3} \therefore x^3 = \frac{8^3}{8} = 64$$

$$\underline{\underline{x = 4}}$$

- 13 P and Q are two fixed points on the surface of the ocean which are 6.0m apart.

An ocean wave travels in the direction P to Q.

The wave has a frequency of 0.50Hz and travels at a constant speed.

A wave peak passes Q at time $t = 0$ s.

The next wave peak travelling towards Q passes P at time $t = 0.80$ s.

What is the speed of the wave?

A 2.1 ms^{-1}

B 3.4 ms^{-1}

☒ C 5.0 ms^{-1}

D 7.5 ms^{-1}

E 20 ms^{-1}

Q

P

$$\text{Frequency} = 0.5 \text{ Hz}$$

$$\begin{aligned} \text{Wavelength} &= \text{Distance between peaks} \\ &= 6 + 0.8v \end{aligned}$$

$$v = 0.5(6 + 0.8v) = 3 + 0.4v$$

$$0.6v = 3 \therefore v = \underline{\underline{5}}$$

- 14 In a sale, all prices are reduced by 25%.

A customer calculates the pre-sale price of a bicycle incorrectly by increasing the marked sale price by 25%.

The customer's calculated pre-sale price is incorrect by £15.

What is the correct pre-sale price of the bicycle?

A £180

B £195

C £210

D £225

☒ E £240

$$\begin{aligned} \text{Marked sale price} &= x \\ \frac{x}{0.75} - 1.25x &= 15 \\ \frac{4x}{3} - \frac{5x}{4} &= 15 \\ \frac{x}{12} &= 15 \therefore x = 15 \times 12 = 180 \\ \text{Pre-sale price} &= \frac{180}{0.75} = 240 \end{aligned}$$

- 15 A parachutist of mass 80.0 kg drops from a plane travelling at 40.0 m s⁻¹, 2000 m above the Earth's surface.

The parachutist hits the ground at a speed of 5.00 m s⁻¹.

How much work is done by the parachutist against drag forces during the fall?

(Take the Earth's gravitational field strength to be 10.0 N kg⁻¹.)

A 1 535 000 J

B 1 624 000 J

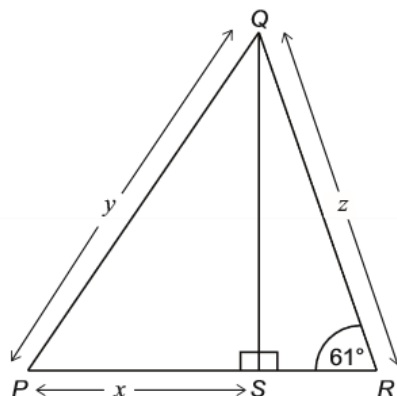
C 1 649 000 J

☒ D 1 663 000 J

E 1 726 000 J

$$\begin{aligned} \Delta KE &= \frac{1}{2}(80)(40^2) - \frac{1}{2}(80)(5^2) \\ &= 40(40^2 - 5^2) \\ &= 40(1575) \\ &= 15750 \times 4 = 63000 \text{ J} \\ \Delta GPE &= mgh = 80 \times 10 \times 2000 \\ &= 1600000 \text{ J} \end{aligned}$$

$$1600000 + 63000$$



[diagram not to scale]

In the diagram, QS is perpendicular to PR.

$$PS = x \text{ cm}$$

$$PQ = y \text{ cm}$$

$$QR = z \text{ cm}$$

angle QRS = 61°

PSR is a straight line.

Which one of the following is an expression for the length z , in cm?

A $\sqrt{y^2 + x^2} \sin 61^\circ$

B $\sqrt{y^2 - x^2} \sin 61^\circ$

C $\sqrt{y^2 + x^2} \cos 61^\circ$

D $\sqrt{y^2 - x^2} \cos 61^\circ$

E $\frac{\sqrt{y^2 + x^2}}{\sin 61^\circ}$

☒ F $\frac{\sqrt{y^2 - x^2}}{\sin 61^\circ}$

G $\frac{\sqrt{y^2 + x^2}}{\cos 61^\circ}$

H $\frac{\sqrt{y^2 - x^2}}{\cos 61^\circ}$

$$QS = \sqrt{y^2 - x^2}$$

$$\sin 61 = \frac{\sqrt{y^2 - x^2}}{z}$$

$$z = \frac{\sqrt{y^2 - x^2}}{\sin 61}$$

- 17 A light spring of unstretched length 0.10 m has a spring constant of 20 N m^{-1} . The spring is suspended so that it is vertical and a load of mass 0.050 kg is attached to the end of the spring.

The load is pulled vertically downwards until the length of the spring is 0.30 m. The load is then released.

What is the speed of the load at the instant that the spring returns to its unstretched length?

(gravitational field strength = 10 N kg^{-1} ; assume that resistive forces are negligible)

- A 0 ms^{-1}
 B 4.0 ms^{-1}
 C 6.0 ms^{-1}
 D 12 ms^{-1}
 E 16 ms^{-1}
 F $\sqrt{6} \text{ ms}^{-1}$
 G $\sqrt{12} \text{ ms}^{-1}$
 H $\sqrt{30} \text{ ms}^{-1}$

$$EPE \rightarrow KE + LPE$$

$$\frac{1}{2}(20)(0.2)^2 = \frac{1}{2}(0.05)v^2 + 0.05(10)(0.2)$$

$$0.4 = 0.025v^2 + 0.1$$

$$0.025v^2 = 0.3$$

$$v^2 = 12 \therefore v = \sqrt{12}$$

- 18 Two vertices of a square are at (1, 1) and (3, 5).

What is the difference between the perimeters of the largest and smallest possible squares that can be drawn with these points as two of their vertices?

- A 0
 B $4\sqrt{3}(2-\sqrt{2})$
 C $4\sqrt{3}(\sqrt{2}-1)$
 D $4\sqrt{5}(2-\sqrt{2})$
 E $4\sqrt{5}(\sqrt{2}-1)$
 F $4\sqrt{13}(2-\sqrt{2})$
 G $4\sqrt{13}(\sqrt{2}-1)$
 H $4\sqrt{3}\sqrt{5}(2-\sqrt{2})$

Largest:

$$\text{side length} = |\vec{AB}| = \sqrt{2^2 + 4^2}$$

$$= \sqrt{20}$$

$$= 2\sqrt{5}$$

$$\text{Perimeter} = 8\sqrt{5}$$

Smallest:

$$\text{Diagonal} = 2\sqrt{5} \therefore \text{side length} = \sqrt{2} \sqrt{5}$$

$$\text{Perimeter} = 4\sqrt{2} \sqrt{5}$$

- 19 A rocket travelling in space is burning its fuel at a constant rate. By expelling the burnt fuel through a nozzle, the engine is applying a constant force to the rocket.

What is happening to the magnitude of the acceleration of the rocket?

(A) It is increasing at an increasing rate.

Constant F .

B It is increasing at a constant rate.

Mass decreasing at a constant rate.

C It is increasing at a decreasing rate.

D It is not changing.

E It is decreasing at an increasing rate.

$$a = \frac{F}{m}$$

F It is decreasing at a constant rate.

Let rate of mass decrease = 0.1 m/s

G It is decreasing at a decreasing rate.

$$a_0 = \frac{F}{m_0} \quad a_1 = \frac{F}{0.9m_0} \quad a_2 = \frac{F}{0.8m_0}$$

$$a_2 - a_1 > a_1 - a_0$$

- 20 The quadratic equation $2x^2 - px - 4 = 0$, where p is a positive constant, has two solutions that differ by 6.

What is the value of p ?

A 2

(B) $4\sqrt{7}$

C 12

D $4\sqrt{11}$

E $4\sqrt{34}$

F $6\sqrt{30}$

$$x = \frac{p \pm \sqrt{p^2 + 32}}{4}$$

$$\frac{p + \sqrt{p^2 + 32}}{4} - \frac{p - \sqrt{p^2 + 32}}{4} = 6$$

$$\frac{2\sqrt{p^2 + 32}}{4} = 6$$

$$\sqrt{p^2 + 32} = 12 \therefore p^2 = 144 - 32 = 112$$

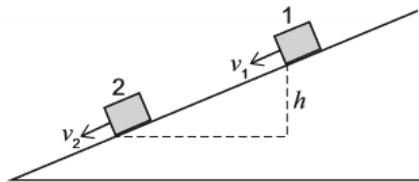
$$p = +\sqrt{112} = 4\sqrt{7}$$

PART B Advanced Mathematics and Advanced Physics

- 21 A block of mass m slides down a rough slope.

At position 1 the velocity of the block is v_1 .

At position 2, which is a vertical distance h below position 1, the velocity of the block is v_2 .



Which expression gives the work done against friction by the block as it slides from position 1 to position 2?

(gravitational field strength = g ; assume that air resistance is negligible)

A $mgh + \frac{1}{2}m(v_2^2 - v_1^2)$

B $mgh - \frac{1}{2}m(v_2^2 - v_1^2)$

C $mgh + \frac{1}{2}m(v_2 - v_1)^2$

D $mgh - \frac{1}{2}m(v_2 - v_1)^2$

E $\frac{1}{2}m(v_2^2 - v_1^2) - mgh$

F $\frac{1}{2}m(v_1^2 - v_2^2) - mgh$

G $\frac{1}{2}m(v_2 - v_1)^2 - mgh$

H $\frac{1}{2}m(v_1 - v_2)^2 - mgh$

$$\frac{1}{2}mv_1^2 + mgh = W_{\text{friction}} + \frac{1}{2}mv_2^2$$

$$\therefore W_{\text{friction}} = mgh + \frac{1}{2}m(v_1^2 - v_2^2)$$

- 22 $(x - 1)$ and $(x - 2)$ are both factors of $x^4 + ax^3 + bx^2 - 12x + 4$

What are the values of a and b ?

A $a = -6$ and $b = -23$

B $a = -6$ and $b = 13$

C $a = 6$ and $b = -11$

D $a = 6$ and $b = 1$

$$f(1) = 0$$

$$1^4 + a(1^3) + b(1^2) - 12(1) + 4 = 0$$

$$1 + a + b - 12 + 4 = 0$$

$$a + b = 7$$

$$f(2) = 0$$

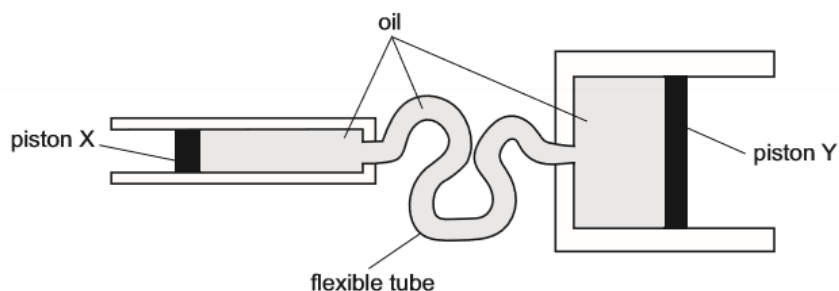
$$16 + 8a + 4b - 24 + 4 = 0$$

$$8a + 4b = 4 \therefore 2a + b = 1$$

18

$$a = -6, b = 13$$

- 23 The braking system of a car includes two cylinders containing an incompressible oil, linked by a flexible tube that also contains oil. There is a freely moving piston in each cylinder. One piston is labelled X, and the other is labelled Y in the diagram.



When the driver presses on the brake pedal, a force is exerted on piston X. The pressure produced in the oil by this force is transmitted through the oil so that it causes a force to act on piston Y. This presses the brakes against the moving parts.

The diameter of piston X is 4.0 cm. The diameter of piston Y is 12.0 cm.

The driver exerts a force of 36.0 N on piston X and it moves a distance of 5.4 cm to the right.

What is the resultant force on piston Y and how far does it move along the cylinder?

	force on piston Y / N	distance moved by piston Y / cm
A	4.0	0.60
B	4.0	48.6
C	12.0	1.80
D	12.0	16.2
E	108	1.80
F	108	16.2
G	324	0.60
H	324	48.6

Same Pressure

$$\therefore \frac{F_x}{A_x} = \frac{F_y}{A_y}$$

$$\frac{36}{\pi \times 4^2 / 4} = \frac{F_y}{\pi \times 12^2 / 4}$$

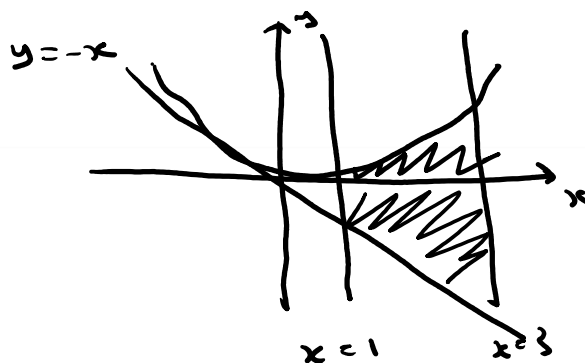
$$F_y = 36 \times \frac{144}{16} \\ = 36 \times 9 = 324 \text{ N}$$

$$W_{d_x} = W_{d_y} \therefore 36 \times 5.4 = 324 \times d_y$$

$$d_y = \frac{5.4}{9} = 0.6$$

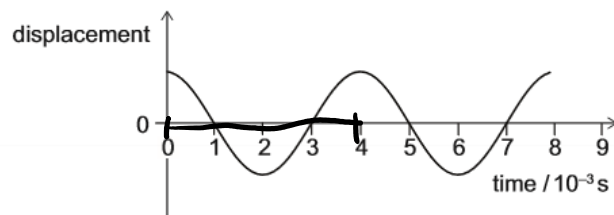
- 24 What is the area of the region enclosed between the curve $y = \frac{1}{2}x^2$, the line $y = -x$, and the lines $x = 1$ and $x = 3$?

- A $\frac{1}{3}$
B 2
C 4
D 6
☒ E $\frac{25}{3}$
F $\frac{28}{3}$



$$\begin{aligned} \text{Area} &= \int_1^3 \frac{1}{2}x^2 dx + \frac{1}{2}(1+3)(2) \\ &= \left[\frac{x^3}{6} \right]_1^3 + 4 = \frac{26}{6} + 4 = \frac{25}{3} \end{aligned}$$

- 25 The graph shows how the displacement due to a wave in air varies with time.



The speed of the wave in air is 300 m s^{-1} .

$$f_{\text{air}} = \frac{1}{T}$$

The wave now travels into water.

$$\frac{\text{wave speed in air}}{\text{wave speed in water}} = 0.2$$

$$T = 0.004 \text{ s}$$

$$f = 250 \text{ Hz}$$

What is the wavelength of the wave in water?

A $\frac{1}{6} \text{ m}$

B $\frac{2}{9} \text{ m}$

C $\frac{5}{6} \text{ m}$

D $\frac{9}{10} \text{ m}$

E $\frac{10}{9} \text{ m}$

F $\frac{6}{5} \text{ m}$

G $\frac{9}{2} \text{ m}$

H 6 m

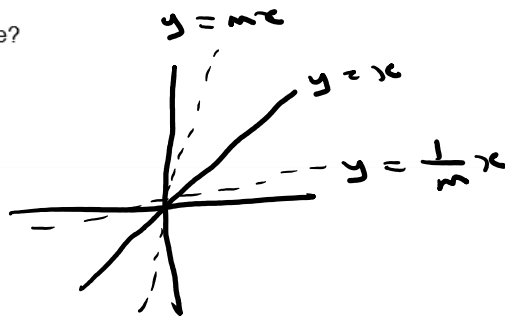
$$\lambda_{\text{air}} = \frac{300}{250} = 1.2 \text{ m}$$

$$\lambda_{\text{water}} = 5 \lambda_{\text{air}} = 6 \text{ m}$$

- 26 A line with non-zero gradient m is reflected in the line $y = x$

What is the gradient of the reflected line?

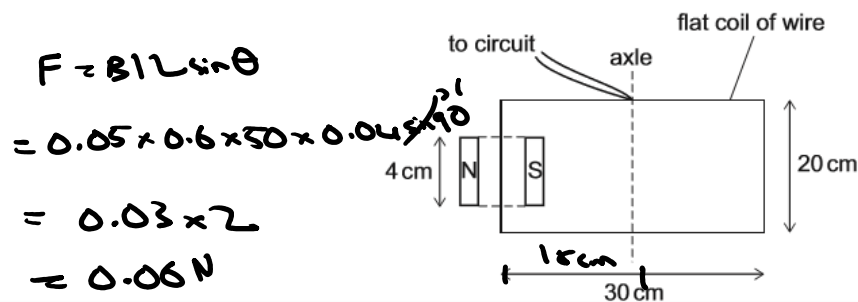
- A m
- B $-m$
- ☒ C $\frac{1}{m}$
- D $-\frac{1}{m}$



- 27 A flat rectangular coil of wire with sides of length 30 cm and 20 cm is freely pivoted about an axle. The axle passes through the middle of the sides of length 30 cm.

Part of the coil is between the poles of a U-shaped magnet as shown in the diagram. The poles are 4.0 cm long. The magnetic field can be considered uniform between the poles, and zero elsewhere.

The coil is connected to a power supply so that there is a current in it.



Handwritten calculation:

$$M = F \times d = 0.06 \times 15 \text{ cm}$$

$$= 0.9 \text{ N cm}$$

[diagram not to scale]

The current is 0.60 A and the magnetic flux density is 0.050 T. There are 50 turns of wire in the coil.

What is the moment about the axle, in N cm, produced by the magnetic force acting on the coil?

- A 0.018 N cm
- B 0.036 N cm
- C 0.045 N cm
- D 0.90 N cm**
- E 1.8 N cm
- F 2.25 N cm
- G 4.5 N cm

- 28 The sum of the first 20 terms of an arithmetic progression is 50.

The sum of the next 20 terms of the arithmetic progression is -50.

What is the sum of the first 100 terms of the arithmetic progression?

(A) -750

B -350

C -50

D $-\frac{159}{8}$

E $\frac{159}{8}$

F 50

G 350

H 750

$$S_{20} = \frac{20}{2}(2a + 19d) = 50$$

$$2a + 19d = 5$$

$$S_{40} = \frac{40}{2}(2a + 39d) = 50 + -50 = 0$$

$$2a + 39d = 0$$

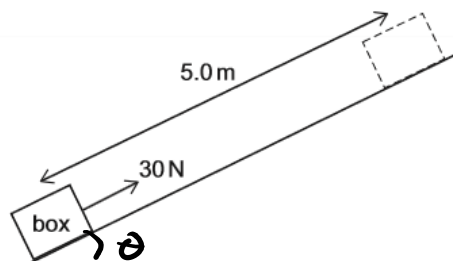
$$20d = -5 \therefore d = -\frac{1}{4}$$

$$a = \frac{5 + \frac{19}{4}}{2} = \frac{39}{8}$$

$$S_{100} = 50\left(\frac{39}{4} + 99\left(-\frac{1}{4}\right)\right) = 50(-15) = \underline{\underline{-750}}$$

- 29 A box of mass 3.0 kg is pulled a distance 5.0 m directly up a smooth slope by a constant applied force of 30 N acting parallel to the slope.

The initial speed of the box is 3.0 m s^{-1} and the final speed is 7.0 m s^{-1} . Its acceleration is constant.



What is the component of the weight acting down the slope?

(gravitational field strength = 10 N kg^{-1} ; air resistance is negligible)

- A 12 N
- B 18 N**
- C 22 N
- D 28 N
- E 29 N
- F 42 N
- G 90 N

$$30 - mg \sin \theta = ma$$

$$v^2 = u^2 + 2as \therefore a = \frac{7^2 - 3^2}{2(5)} = \frac{40}{10} = 4\text{ m s}^{-2}$$

$$30 - mg \sin \theta = 3(4) = 12 \therefore mg \sin \theta = \underline{\underline{18\text{ N}}}$$

- 30 The line L with equation $y = mx + c$, where $m > 0$ and $c \geq 0$, passes through the point (2, 4).

A line is drawn through the point (2, 4) perpendicular to L.

The triangle enclosed between the two lines and the y-axis has area 5 square units.

What is the **larger** of the two possible values of m ?

A -0.5

B 0.5

C 1.25

(D) 2

E 5



$$L \rightarrow L_1 = 2m + c_0$$

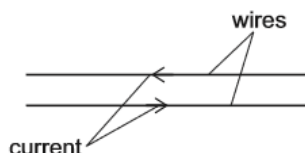
$$\frac{1}{2}(c_1 - c_0)(2) = 5$$

$$\text{Perpendicular} \rightarrow L_2 = -\frac{2}{m} + c_1 \quad c_1 - c_0 = 5$$

$$\therefore L_1 + \frac{2}{m} - L_2 + 2m = 5 \rightarrow 2m^2 - 5m + 2 = 0$$

$$(2m - 1)(m - 2) = 0$$

- 31 Electrical power is supplied through a dc transmission line that consists of two metal wires. Each wire is 8.0 km long and has a cross-sectional area of 1.0 cm^2 .



The resistivity of the metal from which the wires are made is $2.5 \times 10^{-7} \Omega \text{ m}$.

Electrical power is transmitted to the transmission line at a potential difference of 24 000 V.

At what rate is energy being wasted as heat in the wires when the power supplied to the transmission line is 120 kW?

A 0.40 W

B 0.80 W

C 1.6 W

D 250 W

E 500 W

(F) 1000 W

G $1.44 \times 10^7 \text{ W}$

H $5.76 \times 10^7 \text{ W}$

$$R_{\text{for 1 wire}} = \frac{\rho L}{A} = \frac{2.5 \times 10^{-7} \times 8000}{1 \times 10^{-4}}$$

$$= \frac{2.5 \times 8}{1} = 20 \Omega$$

$$VI = 120000 \therefore I = \frac{120000}{24000} = 5 \text{ A}$$

$$\text{Total P loss} = I^2 R_{\text{tot}} = 5^2 (20) \times 2 = \underline{\underline{1000 \text{ W}}}$$

- 32 P and Q are two different geometric progressions.

The 3rd term of each geometric progression is 4.

$$S_{\infty P} - S_{\infty Q} = 16\sqrt{2}$$

The 5th term of each geometric progression is 2.

What is the modulus of the difference between the sums to infinity of P and Q ?

A 0

$$ar^2 = 4$$

B 8

$$ar^4 = 2$$

C $8\sqrt{2}$

$$\therefore r^2 = \frac{1}{2}, r = \pm \frac{1}{\sqrt{2}}$$

D 16

☒ E $16\sqrt{2}$

$$P \rightarrow r = \frac{1}{\sqrt{2}}, a = 8 \rightarrow S_{\infty} = \frac{8}{1 - \frac{1}{\sqrt{2}}} = \frac{8\sqrt{2}}{\sqrt{2} - 1}$$

F 32

$$Q \rightarrow r = -\frac{1}{\sqrt{2}}, a = 8 \rightarrow S_{\infty} = \frac{8}{1 + \frac{1}{\sqrt{2}}} = \frac{8\sqrt{2}}{\sqrt{2} + 1}$$

G $32\sqrt{2}$

$$S_{\infty P} = \frac{8\sqrt{2}(\sqrt{2}+1)}{1} = 16 + 8\sqrt{2} \quad S_{\infty Q} = \frac{8\sqrt{2}(\sqrt{2}-1)}{1} = 16 - 8\sqrt{2}$$

- 33 A tennis ball travelling at 24.0 m s^{-1} is hit by a racket. As a result of the impact, the ball returns back along its original path having undergone a change in velocity of 48.0 m s^{-1} . The acceleration of the ball whilst in contact with the racket is constant with magnitude 6000 m s^{-2} .

What is the total distance travelled by the ball whilst in contact with the racket?

A 0.00 cm

$$v^2 = u^2 + 2as$$

B 4.80 cm

$$0 = 24^2 + 2(6000)(s)$$

☒ C 9.60 cm

$$s = \frac{24^2}{12000} \rightarrow \text{From } 24 \text{ m/s to } 0$$

D 14.4 cm

E 19.2 cm

$$S_{\text{tot}} = \frac{2 \times 24^2}{12000} = \frac{24^2}{6000} = 4 \times 0.024 = \underline{\underline{0.096 \text{ m}}}$$

34 The curve

$$y = x^3 + 3\sqrt{5}px^2 + 3px + 13$$

has two distinct turning points.

What are all the possible values of p ?

☒ A $p < 0, p > 0.2$

B $p \leq 0, p \geq 0.2$

C $0 < p < 0.2$

D $0 \leq p \leq 0.2$

E $p < 0, p > 1.2$

F $p \leq 0, p \geq 1.2$

G $0 < p < 1.2$

H $0 \leq p \leq 1.2$

$$\frac{dy}{dx} = 3x^2 + 6\sqrt{5}px + 3p$$

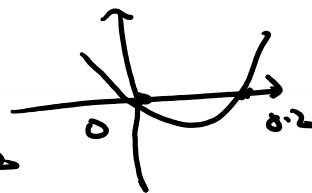
$$\text{discriminant} > 0$$

$$(6\sqrt{5}p)^2 - 4(3)(3p) > 0$$

$$180p^2 - 36p > 0$$

$$36p(5p - 1) > 0$$

$$\therefore p < 0 \text{ or } p > 0.2$$



35 The diagram shows the relative positions of two identical light springs, both in equilibrium.

The springs are supporting loads of mass 0.20 kg and 1.0 kg as shown.

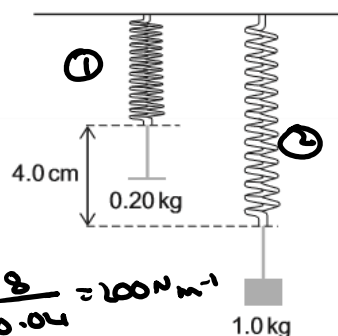
$$\textcircled{1} F = kx$$

$$2 = kx_0$$

$$\textcircled{2} 10 = kx_1$$

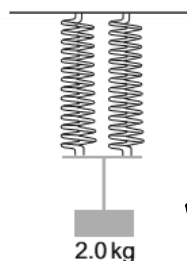
$$x_1 - x_0 = 0.04$$

$$\frac{10}{k} - \frac{2}{k} = 0.04 \therefore k = \frac{8}{0.04} = 200 \text{ N m}^{-1}$$



[diagram not to scale]

The same two springs are now connected in parallel, supporting a 2.0 kg mass as shown.



[diagram not to scale]

Springs in Parallel \therefore
overall $k = 2 \times 200$
 $= 400 \text{ N m}^{-1}$

$$E_{\text{el}} = \frac{1}{2}Fx = \frac{1}{2}F\left(\frac{F}{k}\right)$$

$$= \frac{F^2}{2k} = \frac{20^2}{800}$$

$$= \underline{\underline{0.5 \text{ J}}}$$

In this parallel arrangement, what is the total strain energy in the springs?

(gravitational field strength = 10 N kg^{-1} ; assume that the springs obey Hooke's law)

A 0.25 J

B 0.40 J

C 0.50 J

D 1.0 J

E 25 J

F 40 J

G 50 J

H 100 J

- 36 Find the number of solutions of the equation

$$14 \cos^3 x + 10 \sin^2 x \cos x = 13 \cos x$$

in the range $-2\pi \leq x \leq 2\pi$

A 4

B 6

C 8

D 10

E 12

F 14

$$14 \cos^3 x + 10(1 - \cos^2 x) \cos x = 13 \cos x$$

$$4 \cos^2 x - 3 \cos x = 0$$

$$\cos x (4 \cos^2 x - 3) = 0$$

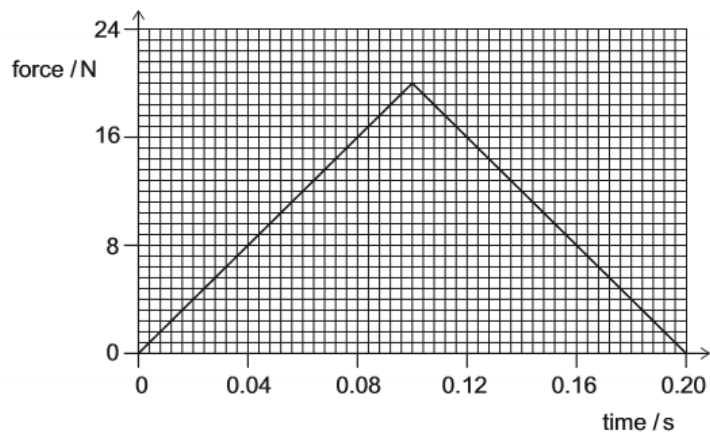
$$\cos x = 0 \quad \text{or} \quad \cos x = \pm \frac{\sqrt{3}}{2} \rightarrow 4 \text{ sols each}$$

4 solutions



- 37 An object of mass 2.5 kg is at rest at time = 0 s. A resultant force acts on the object in a constant direction.

The magnitude of the resultant force acting on the object varies with time as shown by the graph.



What is the kinetic energy of the object at time = 0.20 s?

- A 0 J
- B 0.80 J**
- C 1.0 J
- D 1.6 J
- E 2.0 J
- F 3.2 J
- G 6.4 J

Impulse = Area under graph
 $= \frac{1}{2} \times 0.2 \times 20$
 $= 2 \text{ Ns}$

$\Delta p = 2 \text{ kg ms}^{-1}$
 $2.5(v_1 - 0) = 2 \therefore v = 0.8 \text{ ms}^{-1}$
 $E_k = \frac{1}{2} \times 2.5 \times 0.64 = \frac{1}{2} \times \frac{25}{10} \times \frac{16}{10} = \frac{8}{10}$

- 38 Find the product of the real roots of the equation

$$(\log_{10} x^2)^2 + \log_{10} x = 3$$

- A $10^{-\frac{3}{2}}$
B 10^{-1}
C $10^{-\frac{1}{2}}$
D $10^{-\frac{1}{4}}$
E $10^{\frac{3}{5}}$
F 10^1

$$(2 \log_{10} x)^2 + \log_{10} x = 3$$

$$\text{Let } v = \log_{10} x$$

$$4v^2 + v = 3$$

$$4v^2 + v - 3 = 0$$

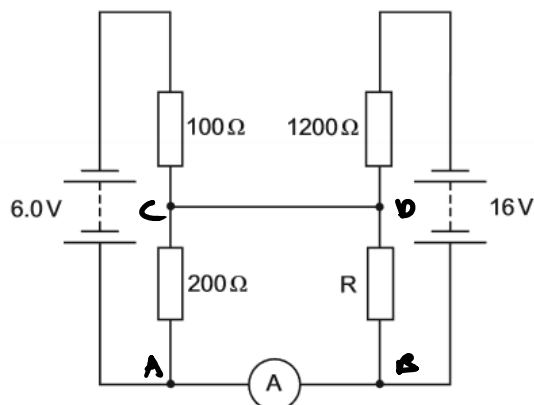
$$(4v - 3)(v + 1) = 0$$

$$\log_{10} x = \frac{3}{4} \text{ or } \log_{10} x = -1$$

$$x = 10^{3/4} \text{ or } 10^{-1}$$

$$10^{3/4} \times 10^{-1} = 10^{3/4-1} = 10^{-1/4}$$

- 39 In the following circuit, the ammeter records a current of zero.



What is the resistance of resistor R?

- A 0Ω
- B 200Ω
- C 300Ω
- D 400Ω**
- E 600Ω
- F 1200Ω
- G 1800Ω
- H 2400Ω

$$V_A = V_B \text{ for } 0 \text{ current}$$

$$\therefore V_C = V_D$$

$$\frac{200}{200+100} \times 6 = \frac{R}{R+1200} \times 16$$

$$\frac{2}{3} \times 6 = \frac{R}{R+1200} \times 16$$

$$\frac{R}{R+1200} = \frac{4}{16} = \frac{1}{4} \therefore R+1200=4R$$

$$R = \frac{1200}{3} = 400\Omega$$

- 40 Find the maximum value of the gradient of the curve with equation

$$y = 2 - 4x + 4x^{\frac{3}{2}} - x^2$$

where $x > 0$

A -4

B $-\frac{8}{9}$

☒ C $\frac{1}{2}$

D 2

E 4

$$\frac{dy}{dx} = -4 + 6x^{\frac{1}{2}} - 2x$$

$$\frac{d^2y}{dx^2} = 3x^{-\frac{1}{2}} - 2 = 0$$

$$\frac{3}{\sqrt{x}} = 2$$

$$\sqrt{x} = \frac{3}{2} \Rightarrow x = \frac{9}{4}$$

$$\frac{dy}{dx} = -4 + 6\sqrt{\frac{9}{4}} - \frac{18}{4}$$

END OF TEST

$$= -4 + \frac{18}{2} - \frac{18}{4} = 5 - \frac{9}{2} = \frac{1}{2}$$

**ENGINEERING
ADMISSIONS ASSESSMENT****D564/32****November 2020****60 minutes****SECTION 2****INSTRUCTIONS TO CANDIDATES**

Please read these instructions carefully, but do not open this question paper until you are told that you may do so. This paper is Section 2 of 2.

A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and name.

This paper contains 20 multiple-choice questions. There are no penalties for incorrect responses, only marks for correct answers, so you should attempt **all** 20 questions. Each question is worth one mark.

For each question, choose the **one** option you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

You **must** complete the answer sheet within the time limit.

You can use the question paper for rough working, but **no extra paper** is allowed. Only your responses on the answer sheet will be marked.

Dictionaries and calculators are NOT permitted.

Please wait to be told you may begin before turning this page.

This question paper consists of 22 printed pages and 2 blank pages.



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- 1 Spring P has spring constant 1.0 N cm^{-1} and spring Q has spring constant 3.0 N cm^{-1} .

The two springs are connected in series.

The springs are stretched by 6.0 cm in total.

What is the extension of spring P?

(The springs have negligible mass and obey Hooke's law.)

A 1.5 cm

B 2.0 cm

C 3.0 cm

D 4.0 cm

☒ E 4.5 cm

$$k_{\text{tot}} = \frac{1 \times 3}{1 + 3} = \frac{3}{4} = 0.75 \text{ N cm}^{-1}$$

$$F_P = F_Q = 0.75 \times 6 = 4.5 \text{ N}$$

$$x_P = \frac{4.5 \text{ N}}{1 \text{ N cm}^{-1}} = \underline{\underline{4.5 \text{ cm}}}$$

- 2 A single strand of wire has a radius of 2.0×10^{-4} m and length 15 m. The resistivity of the material from which the wire is made is $4.8 \times 10^{-7} \Omega \text{m}$.

Twelve strands of this wire are connected in parallel to make a cable.

What is the resistance of the cable?

A $\frac{\pi}{2160} \Omega$

B $\frac{\pi}{180} \Omega$

C $\frac{\pi}{15} \Omega$

☒ D $\frac{15}{\pi} \Omega$

E $\frac{180}{\pi} \Omega$

F $\frac{2160}{\pi} \Omega$

$$R_{\text{wire}} = \frac{\rho L}{A} = \frac{4.8 \times 10^{-7} \times 15}{\pi \times (2 \times 10^{-4})^2}$$

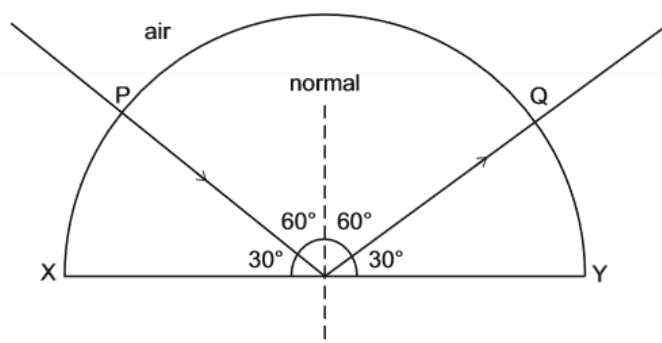
$$= \frac{18 \times 10^{-7}}{\pi \times 4 \times 10^{-8}}$$

$$= \frac{180}{\pi}$$

$$\frac{1}{R_{\text{tot}}} = \frac{1}{180/\pi} + \dots + 12 \text{ times} = \frac{12\pi}{180}$$

$$R_{\text{tot}} = \frac{180}{12\pi} = \frac{15}{\pi}$$

- 3 A ray of light is directed into a semicircular transparent block, entering at P. The direction of the ray is adjusted until it strikes the centre of the flat face XY of the block at the critical angle and reflects to Q as shown.



The length of XY is L .

The speed of light in air is c .

What is the time taken by the light to travel from P to Q in the block?

A $\frac{L\sqrt{3}}{2c}$

B $\frac{L}{c}$

C $\frac{2L}{c\sqrt{3}}$

D $\frac{L\sqrt{3}}{c}$

E $\frac{2L}{c}$

F $\frac{4L}{c\sqrt{3}}$

$$n_b \sin 60 = n_{air} \sin 90$$

$$\frac{n_b}{n_{air}} = \frac{c}{v_b}$$

$$\frac{c}{v_b} \sin 60 = 1$$

$$v_b = \frac{c\sqrt{3}}{2}$$

$$\text{Time} = L \div \frac{c\sqrt{3}}{2}$$

$$= \frac{2L}{c\sqrt{3}}$$

- 4 A solid cube with sides of length 20 cm is made from material with density 2000 kg m^{-3} . The cube is suspended, in equilibrium, from an initially unstretched spring, and this results in the spring gaining strain energy of 3.2 J.

What is the spring constant of the spring?

(gravitational field strength = 10 N kg^{-1} ; the spring obeys Hooke's law)

- A 40 N m^{-1}
B 80 N m^{-1}
C 400 N m^{-1}
D 800 N m^{-1}
☒ E 4000 N m^{-1}
F 8000 N m^{-1}

$$\begin{aligned} \text{Mass} &= 2000 \times 0.2^3 \\ &= 2000 \times 0.008 = 16 \text{ kg} \\ \frac{1}{2}Fx &= 3.2 \\ \frac{160x}{2} &= 3.2 \therefore x = \frac{3.2}{80} = 0.04 \\ k &= \frac{F}{x} = \frac{160}{0.04} = 4000 \end{aligned}$$

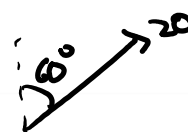
- 5 A projectile is fired upwards from the ground at an angle of 60° to the vertical at a speed of 20 m s^{-1} .

It travels a horizontal distance d and lands with a downwards vertical component of velocity of 4.0 m s^{-1} on ground that is height h above the starting point of the projectile.

What are d and h ?

(gravitational field strength = 10 N kg^{-1} ; assume that air resistance is negligible)

	d / m	h / m
A	$6.0\sqrt{3}$	4.2
B	$6.0\sqrt{3}$	5.8
C	$10\sqrt{3} - 4.0$	4.2
D	$10\sqrt{3} - 4.0$	14.2
E	$10\sqrt{3} + 4.0$	5.8
F	$10\sqrt{3} + 4.0$	14.2
G	$14\sqrt{3}$	4.2
H	$14\sqrt{3}$	5.8



$$\text{vertical comp.} = 20 \cos 60 = 10 \text{ m s}^{-1}$$

$$v = u + at$$

$$-4 = 10 - 10t \quad \therefore t = \frac{14}{10} \text{ s} = 1.4 \text{ s}$$

$$d = 20 \sin 60 \times 1.4$$

$$= 10\sqrt{3} \times 1.4 = \underline{\underline{14\sqrt{3}}}$$

$$v^2 = u^2 + 2as$$

$$4^2 = 10^2 - 20s \quad \therefore s = h = \frac{100 - 16}{20} = 4.2$$

- 6 Diagram 1 shows the positions of nine equally spaced particles in a medium.

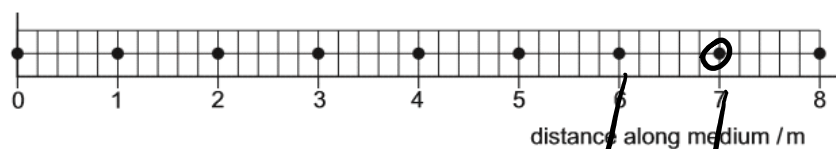


Diagram 1

Diagram 2 shows the positions of the same nine particles, at a particular time, while a longitudinal wave is travelling through the medium.

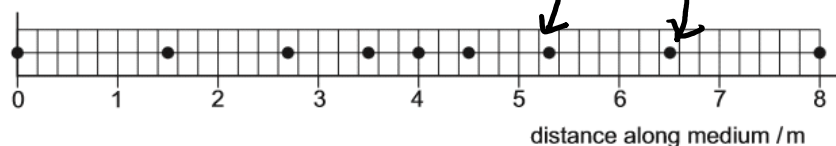


Diagram 2

What is the amplitude of the wave?

- A 0.4 m
- B 0.5 m
- C 0.6 m
- ☒ D 0.7 m
- E 2.0 m
- F 4.0 m
- G 6.0 m
- H 8.0 m

*Amplitude = Max Displacement
from eq. position
Particle at 6 moves 0.7m*

- 7 A spaceship with mass $8.0 \times 10^4 \text{ kg}$ travels at constant velocity and has $1.0 \times 10^{12} \text{ J}$ of kinetic energy.

An external impulse of $8.0 \times 10^7 \text{ kg m s}^{-1}$, lasting for 2.0 s , is applied to the spaceship acting in the opposite direction to the motion of the spaceship.

What is the average rate of loss of kinetic energy of the spaceship during the application of the impulse?

A $9.5 \times 10^{10} \text{ W}$

B $1.8 \times 10^{11} \text{ W}$

C $2.2 \times 10^{11} \text{ W}$

D $3.2 \times 10^{11} \text{ W}$

E $3.6 \times 10^{11} \text{ W}$

F $7.2 \times 10^{11} \text{ W}$

$$\frac{1}{2} \times 8 \times 10^4 \times v^2 = 1 \times 10^{12}$$

$$v^2 = \frac{10^{12}}{4 \times 10^4} = \frac{10^8}{4}$$

$$v = \frac{10^4}{2} = 5000 \text{ ms}^{-1}$$

$$\text{Impulse} = -8 \times 10^7 = m(-v - 5000)$$

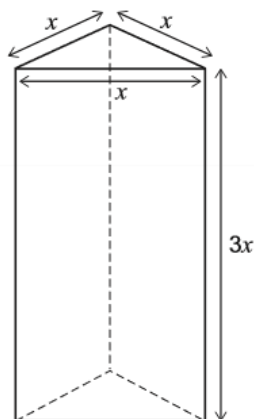
$$8 \times 10^7 = 8 \times 10^4 (v + 5000)$$

$$v = 1000 - 5000 = -4000 \text{ ms}^{-1}$$

$$E_{k \text{ after}} = \frac{1}{2} \times 8 \times 10^4 \times 4000^2 = 4 \times 10^4 \times 16 \times 10^6 = 6.4 \times 10^{11}$$

$$\text{Rate of loss} = 3.6 \times 10^{11} / 2 = 1.8 \times 10^{11} \text{ W}$$

- 8 The diagram shows a solid triangular prism.



$$\begin{aligned}\text{Cross-section Area} &= \frac{1}{2} x^2 \sin 60 \\ &= \frac{x^2 \sqrt{3}}{4} \\ \text{Volume} &= \frac{3x^3 \sqrt{3}}{4}\end{aligned}$$

The sides of the triangular cross section of the prism are of length x .

The height of the prism is $3x$.

The uniform density of the prism is ρ .

The gravitational field strength is g .

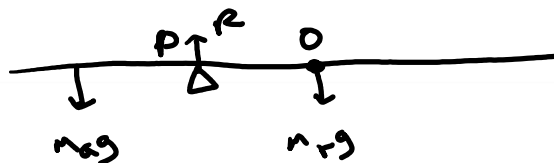
What is the minimum pressure the prism can exert when it rests on level ground?

- A $3\rho g$
- B $3\rho gx$
- C $\frac{\rho g}{4}$
- D $\frac{\rho gx}{4}$
- E $\frac{\sqrt{3}\rho g}{4}$
- ☒ F $\frac{\sqrt{3}\rho gx}{4}$

- 9 An apple of mass m_a is placed on a uniform metre rule with the centre of gravity of the apple at the 10 cm mark. The rule is balanced on a pivot placed at the 35 cm mark.

The apple is replaced with an orange of mass m_o . The rule now balances with the pivot at the 40 cm mark.

What is the ratio $\frac{m_a}{m_o}$?



A $\frac{5}{9}$

B $\frac{4}{5}$

C $\frac{5}{6}$

D $\frac{6}{5}$

E $\frac{5}{4}$

F $\frac{9}{5}$

$$\text{P: } m_o g(15) = m_a g(25)$$

With the orange:

$$m_o g(10) = m_a g(30)$$

$$\frac{15m_o}{10m_a} = \frac{25m_a}{30m_o} \therefore \frac{m_a}{m_o} = \frac{450}{250} = \frac{9}{5}$$

- 10 A cyclist travels at a constant speed of 12 m s^{-1} on level ground. During this time the power needed to maintain a constant speed is 900 W . The total weight of the cyclist and bicycle is 850 N .

The cyclist now cycles up a slope at the same constant speed. The slope is at an angle of 30° to the horizontal.

What is the driving force on the bicycle as it travels up the slope?

(Assume that the magnitude of the resistive forces is constant.)

- A 75 N
B 350 N
C 500 N
D $(425\sqrt{3} - 75) \text{ N}$
E 775 N
F $(425\sqrt{3} + 75) \text{ N}$
G 925 N
- Level Ground $\rightarrow F = \frac{P}{v} = \frac{900}{12} = 75 \text{ N}$
 \therefore Resistive Force $= 75 \text{ N}$
Slope \rightarrow Driving force $= 75 + 850 \sin 30^\circ = 75 + 425 = 500 \text{ N}$

- 11 Three identical resistors can be combined in four different arrangements.

One of the arrangements has a resistance of 18Ω .

A different arrangement has a resistance of 8.0Ω .

What are the resistances of the other two arrangements?

(All three resistors contribute to the total resistance in all arrangements.)

A 2.0Ω and 4.0Ω

B 2.0Ω and 9.0Ω

C 4.0Ω and 12Ω

D 4.0Ω and 36Ω

E 36Ω and 162Ω

F 81Ω and 162Ω



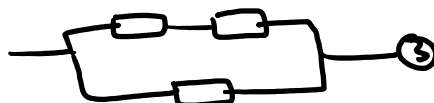
①

$$R_{\text{tot}} = 3R$$



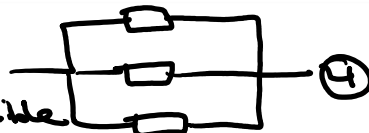
②

$$\frac{3R}{2}$$



③

$$\frac{2R}{3}$$



④

$$\frac{R}{3}$$

If $3R = 18, R = 6$ - Impossible

If $\frac{3R}{2} = 18, R = 12, \frac{2R}{3} = 8 \checkmark \therefore \frac{R}{3} = 4, 3R = 36$

- 12 A $4.0\text{ k}\Omega$ fixed resistor is connected in series with a light dependent resistor (LDR) across a 100 V dc power supply.

The current in the LDR is 5.0 mA .

The intensity of light falling on the LDR now decreases and the voltage across the fixed resistor changes by 50% .

What is the change in the resistance of the LDR as a result of the change in intensity?

A $8.0\text{ k}\Omega$

B $12\text{ k}\Omega$

C $16\text{ k}\Omega$

☒ D $20\text{ k}\Omega$

E $32\text{ k}\Omega$

F $36\text{ k}\Omega$

$$V_{\text{res}} \text{ initially} = 4000 \times 0.005 \\ = 20\text{V}$$

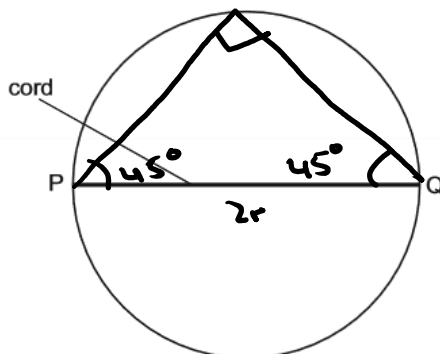
$$R_{\text{LDR}} \uparrow \therefore V_{\text{res}} = 10\text{V}$$

$$I_{\text{res}} = 0.0025\text{ A}$$

$$\text{Initially} \rightarrow \frac{R_{\text{res}}}{R_{\text{res}} + R_{\text{LDR}}} \times 100 = 20 \therefore R_{\text{LDR}} = 16\text{ k}\Omega$$

$$\text{Now, } \frac{R_{\text{res}}}{R_{\text{res}} + R_{\text{LDR}}} \times 100 = 10 \therefore R_{\text{LDR}} = 36\text{ k}\Omega$$

- 13 An elastic cord with spring constant k is fixed to two points P and Q on the diameter of a ring so that the cord is taut but unstretched. The radius of the ring is r .



The midpoint of the cord is then pulled and fixed to a point on the ring halfway between P and Q.

What is the energy stored in the elastic cord?

A $\frac{1}{2}kr^2$

B $2kr^2$

C $\frac{1}{2}(\sqrt{2} - 1)kr^2$

D $2(\sqrt{2} - 1)kr^2$

E $\frac{1}{2}(3 - 2\sqrt{2})kr^2$

F $2(3 - 2\sqrt{2})kr^2$

$$\begin{aligned} \text{New length} &= 2r \cos 45^\circ \times 2 \\ &= 4r \times \frac{\sqrt{2}}{2} = 2r\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{Energy} &= \frac{1}{2}kx^2 \\ &= \frac{1}{2}k(2r\sqrt{2} - 2r)^2 \\ &= \frac{1}{2}k r^2 (8 + 4 - 8\sqrt{2}) \\ &= 2(3 - 2\sqrt{2})kr^2 // \end{aligned}$$

- 14 An object of mass M experiences a resultant force of magnitude F . The force acts in a single horizontal direction with a magnitude that varies with time t according to

$$F = X + Y\sqrt{t}$$

where X and Y are constants.

The object is at rest at $t = 0$.

What is the magnitude of the momentum of the object at time $t = T$?

☒ A $T(X + \frac{2}{3}Y\sqrt{T})$

B $T(X + Y\sqrt{T})$

C $\frac{T}{M}(X + \frac{2}{3}Y\sqrt{T})$

D $\frac{T}{M}(X + Y\sqrt{T})$

E $\frac{Y}{2\sqrt{T}}$

F $\frac{Y}{2M\sqrt{T}}$

$$\begin{aligned}\Delta p &= \int_0^T F \, dt \\ &= \int_0^T (X + Yt^{1/2}) \, dt \\ &= \left[Xt + \frac{2Yt^{3/2}}{3} \right]_0^T \\ &= XT + \frac{2YT\sqrt{T}}{3}\end{aligned}$$

- 15 A trolley of mass 3.0 kg is moving horizontally along a smooth track. Its displacement x from a point at time t is given by the equation:

$$x = 8 + 4t + 2t^2$$

where x is in metres and t is in seconds.

How much work is done on the trolley between times $t = 0$ and $t = 5.0$ s?

A 12 J

B 24 J

C 78 J

D 270 J

☒ E 840 J

F 864 J

G 936 J

$$v = \frac{dx}{dt} = 4 + 4t$$

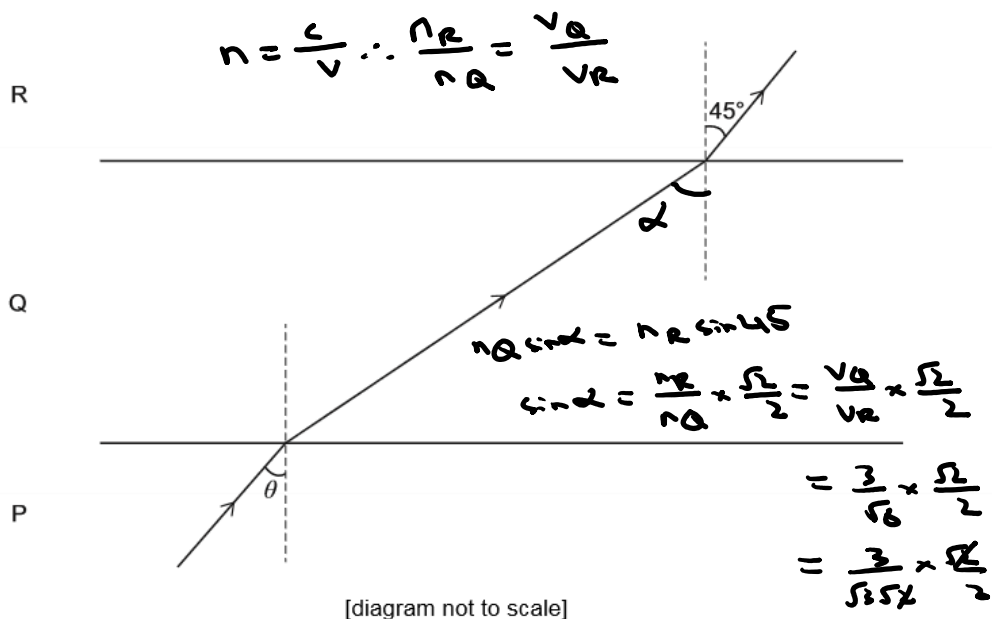
$$v_0 = 4$$

$$v_5 = 24$$

$$\Delta KE = \frac{1}{2}(3)(24^2 - 4^2)$$

$$= \frac{1}{2}(3)(560) = \underline{840 \text{ J}}$$

- 16 The diagram shows a ray of light passing through three media, P, Q and R. The boundaries between the three media are parallel.



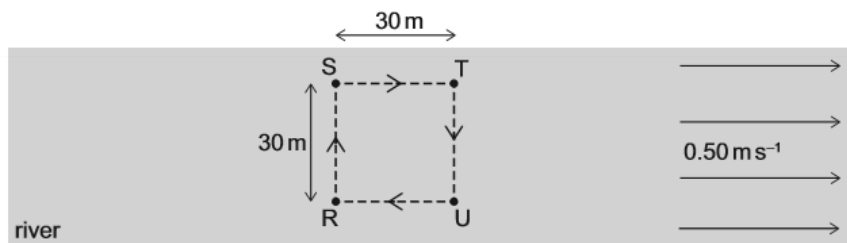
The ratio of the speed of light in medium P to the speed of light in medium Q is $2:\sqrt{5}$

The ratio of the speed of light in medium Q to the speed of light in medium R is $3:\sqrt{6}$ $\therefore \alpha = 60^\circ$

What is the value of $\sin \theta$?

- A $\frac{\sqrt{2}}{2}$
- B $\frac{\sqrt{3}}{2}$
- C $\frac{\sqrt{3}}{6}$
- D $\frac{\sqrt{5}}{5}$
- ☒ E $\frac{\sqrt{15}}{5}$
- F $\frac{\sqrt{15}}{6}$
- $n_P \sin \theta = n_Q \cdot \frac{\sqrt{3}}{2}$
- $\sin \theta = \frac{n_Q}{n_P} \times \frac{\sqrt{3}}{2} = \frac{v_P}{v_Q} \times \frac{\sqrt{3}}{2} = \frac{2}{\sqrt{5}} \times \frac{\sqrt{3}}{2}$
- $= \frac{\sqrt{15}}{5}$

- The swimmer has a constant speed of 1.0 m s^{-1} relative to the water.




A $(60 + 24\sqrt{5})\text{s}$

S → T: Speed = 1.5 ms^{-1} ∴ Time = 20s

- B** $(60 + 40\sqrt{3})\text{s}$

$v \rightarrow R$: Speed = $0.5 \text{ ms}^{-1} \therefore \text{Time} = \underline{\underline{60 \text{ s}}}$

- C** $(80 + 24\sqrt{5})\text{s}$

Time_{RS} = Time_{TU} =>  80V

- D** $(80 + 40\sqrt{3})\text{s}$

$$v^2 + 0.5^2 = 1^2$$

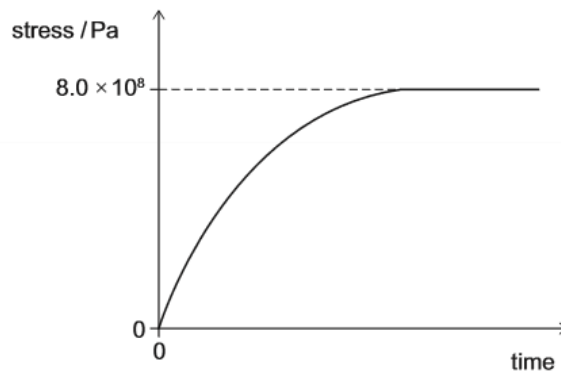
$$v^2 = 3/4 \quad v = \sqrt{3}/2$$

- E** 120 s

$$T = \frac{30 \div \frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}} = \frac{60}{\sqrt{3}} = 20\sqrt{3} \quad \therefore T_{\text{tot}} = 2 \times 20\sqrt{3} = 40\sqrt{3}$$

- F** 140 s

- 18 The stress in a steel cable increases with time and is then maintained at a constant value, as shown. The wire does not reach its limit of proportionality.



The table shows properties of the steel used in the cable and the dimensions of the cable.

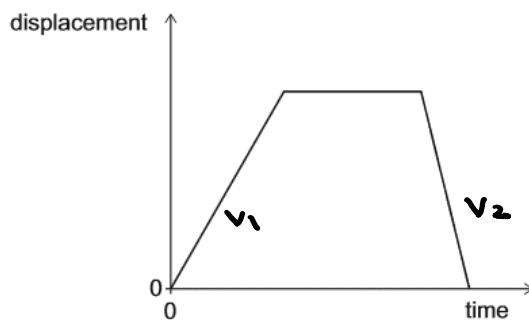
length / m	cross-sectional area / m ²	Young modulus / Pa
4.0	2.0×10^{-4}	2.0×10^{11}

How much work was done to stretch the cable?

- A 320 J
- B 1.28 kJ**
- C 2.56 kJ
- D 320 kJ
- E 640 kJ
- F 1.60 MJ
- G 6.40 MJ

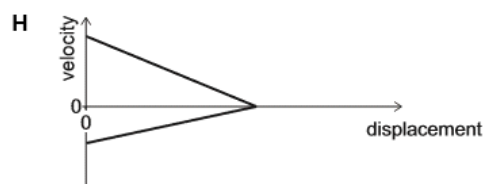
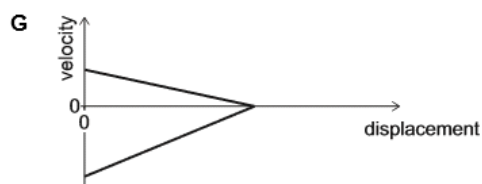
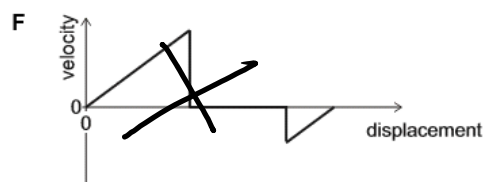
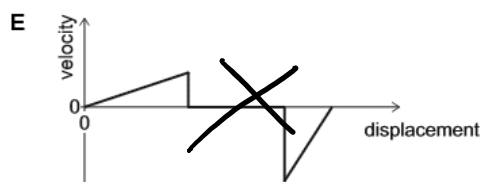
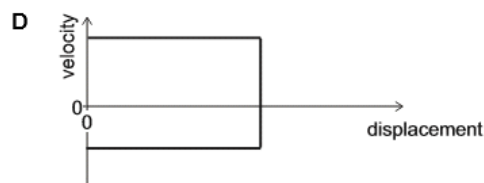
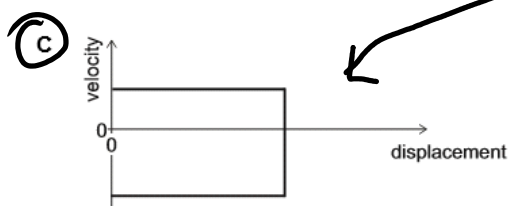
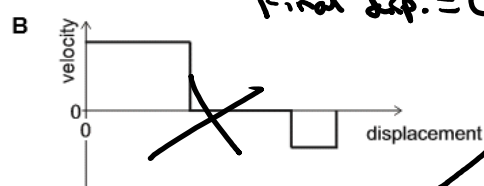
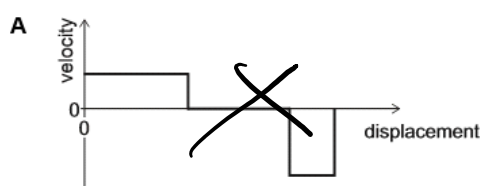
$$\begin{aligned}
 \text{Work Done} &= \frac{1}{2} \times F \times x \rightarrow x = \frac{FL}{AE} \\
 &= \frac{1}{2} \times 8 \times A \times \frac{6L}{E} \\
 &= \frac{\frac{1}{2} \times (8 \times 10^8)^2 \times 2 \times 10^{-4} \times 4}{2 \times 10^{11}} \\
 &= \frac{64 \times 10^8 \times 10^{-4} \times 2}{10^{11}} = 128 \times 10 = \underline{1280 \text{ J}}
 \end{aligned}$$

- 19 The following graph shows how the displacement of an object travelling along a straight, horizontal track varies with time.



Which graph shows the velocity of this object against displacement?

$|v_2| > |v_1|$
Final $\Delta x = 0$



- 20 A cell has emf E and internal resistance r that varies with current I according to:

$$r = kI^2$$

where k is a constant.

A variable resistor is connected to the terminals of the cell. The resistance of the variable resistor is adjusted.

Which expression gives the resistance of the variable resistor, in terms of k and E , that causes maximum power dissipation in it?

A $3\left(\frac{kE^2}{2}\right)^{\frac{1}{3}}$

B $3\left(\frac{kE^2}{4}\right)^{\frac{1}{3}}$

C $3\left(\frac{kE^2}{9}\right)^{\frac{1}{3}}$

D $3\left(\frac{kE^2}{16}\right)^{\frac{1}{3}}$

E $(2kE^2)^{\frac{1}{3}}$

F $(4kE^2)^{\frac{1}{3}}$

G $(9kE^2)^{\frac{1}{3}}$

H $(16kE^2)^{\frac{1}{3}}$

$$E = Ir + IR_{\text{var}} = Ir + IR$$

$$R = \frac{E}{I} - r = \frac{E}{I} - kI^2$$

$$P = I^2 R = EI - kI^3$$

$$\frac{dP}{dI} = E - 3kI^2 = 0 \text{ for max power}$$

$$\therefore E = 3kI^2 \Rightarrow I = \left(\frac{E}{3k}\right)^{\frac{1}{2}}$$

$$R = \frac{E}{I} - kI^2 = E \left(\frac{3k}{E}\right)^{\frac{1}{2}} - k \left(\frac{E}{3k}\right)^{\frac{1}{2}}$$

$$= (3kE)^{\frac{1}{2}} - \left(\frac{kE^2}{16}\right)^{\frac{1}{3}}$$

$$= \left(\frac{64}{16}kE^2\right)^{\frac{1}{3}} - \left(\frac{1}{16}kE^2\right)^{\frac{1}{3}}$$

$$= \left(\frac{kE^2}{16}\right)^{\frac{1}{3}} (4^{\frac{1}{3}} - 1^{\frac{1}{3}}) = (4-1) \left(\frac{kE^2}{16}\right)^{\frac{1}{3}}$$

$$= 3 \left(\frac{kE^2}{16}\right)^{\frac{1}{3}}$$

END OF TEST